

NEUTRINO MASS AND NEUTRINOLESS DOUBLE BETA DECAY

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ABSTRACT

The motivation for the search for $0\nu\beta\beta$ decay is briefly reviewed. It is stressed that the exchange of light Majorana neutrinos is not the only possible mechanism of the decay. The link between lepton number and lepton flavor violation is described and its role in elucidating the $0\nu\beta\beta$ -decay mechanism is discussed. The main topic of the talk is the evaluation of the nuclear matrix elements and their uncertainty. Various physics effects that influence the value of the matrix elements are described and the results of the two main methods, the quasiparticle random phase approximation and the nuclear shell model, are compared.

1. Introduction

In the last decade neutrino oscillation experiments have convincingly and triumphantly shown, using both the natural and manmade neutrino sources, that neutrinos have a finite mass and that the lepton flavor is not a conserved quantity. These results opened the door to what is often called the “Physics Beyond the Standard Model”. In other words, accommodating these findings into a consistent scenario requires generalization of the Standard Model of electroweak interactions that postulates that neutrinos are massless and that consequently lepton flavor, and naturally, also the total lepton number, are conserved quantities.

The present results of the oscillation experiments are summarized in Fig. 1 that shows the decomposition of the flavor eigenstates neutrinos ν_e, ν_μ and ν_τ into the mass eigenstates ν_1, ν_2 and ν_3 . An upper limit on the masses of all active neutrinos $\sim 2 - 3$ eV can be derived from the combination of analysis of the tritium beta-decay experiments and the neutrino oscillation experiments. Combining these constraints, masses of at least two (out of the total of three active) neutrinos are bracketted by $10 \text{ meV} \leq m_\nu \leq 2 - 3 \text{ eV}$.

Therefore, neutrino masses are six or more orders of magnitude smaller than the masses of the other fermions. Moreover, the pattern of masses, i.e. the mass ratios of neutrinos, is rather different (even though it remains largely unknown) than the pattern of masses of the up- or down-type quarks or charged leptons. All of these

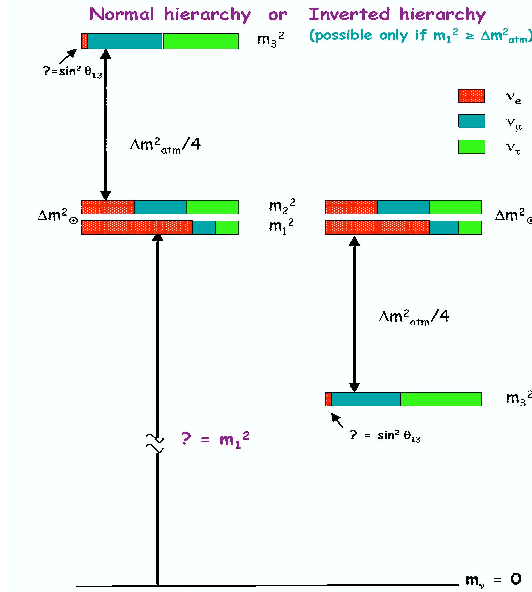


Figure 1: Schematic illustration of the decomposition of the neutrino mass eigenstates ν_i in terms of the flavor eigenstates. The two hierarchies cannot be, at this time, distinguished. The small admixture of ν_e into ν_3 is an upper limit, and the mass square of the neutrino ν_1 , the quantity m_1^2 , remains unknown.

facts suggest that, perhaps, the origin of the neutrino mass is different than the origin (which is still not well understood) of the masses of the other charged fermions.

The smallness of the neutrino masses can be understood following the finding of Weinberg ¹⁾ who pointed out almost thirty year ago that there exists only one lowest order (dimension 5, suppressed by only one inverse power of the corresponding high energy scale Λ) gauge-invariant operator given the content of the standard model. After the spontaneous symmetry breaking when the Higgs acquires vacuum expectation value that operator represents the neutrino Majorana mass which violates the total lepton number conservation law by two units,

$$\mathcal{L}^{(M)} = \frac{C^{(5)}}{\Lambda} \frac{v^2}{2} (\bar{\nu}^c \nu) + h.c. \quad , \quad (1)$$

where $v \sim 250$ GeV and $C^{(5)}$ is expected to be of the order of unity. For sufficiently large scale Λ neutrinos masses are arbitrarily small.

The most popular explanation of the smallness of neutrino mass is the see-saw mechanism, which is also roughly thirty years old ²⁾. In it, the existence of heavy right-handed neutrinos N_R is postulated, and by diagonalizing the corresponding mass matrix one arrives at the formula

$$m_\nu = \frac{m_D^2}{M_N} \quad (2)$$

where the Dirac mass m_D is expected to be a typical charged fermion mass and M_N is the Majorana mass of the heavy neutrinos N_R . Again, the small mass of the standard neutrino is related to the large mass of the heavy right-handed partner. Requiring that m_ν is of the order of 0.1 eV means that M_N (or Λ) is $\sim 10^{14-15}$ GeV, i.e. near the GUT scale. That makes this template scenario particularly attractive.

Clearly, one cannot reach such high energy scale experimentally. But these scenarios imply that neutrinos are Majorana particles, and consequently that the total lepton number should not be conserved. Hence the tests of the lepton number conservation acquires a fundamental importance.

There are various ways to test whether the total lepton number is conserved or not. Examples of the potentially lepton number violating (LNV) processes with important limits are

$$\begin{aligned} (Z, A) &\rightarrow (Z + 2, A) + 2e^-; \text{ halflife } > 10^{25} \text{ years} \\ \mu^- + (Z, A) &\rightarrow e^+ + (Z - 2, A); \text{ exp. branching ratio } \leq 10^{-12}, \\ K^+ &\rightarrow \mu^+ \mu^+ \pi^-; \text{ exp. branching ratio } \leq 3 \times 10^{-9}, \\ \bar{\nu}_e \text{ emission from the Sun;} &\text{ exp. branching ratio } \leq 10^{-4}. \end{aligned} \quad (3)$$

However, detailed analysis suggests that the study of the $0\nu\beta\beta$ decay, the first on the list above, is by far the most sensitive test of LNV. In simple terms this is caused by the amount of tries one can make. A 100 kg $0\nu\beta\beta$ decay source contains $\sim 10^{27}$ nuclei that can be observed for a long time (several years). This can be contrasted with the possibilities of first producing muons or kaons, and then searching for the unusual decay channels. The Fermilab accelerators, for example, produce $\sim 10^{20}$ protons on target per year in their beams and thus correspondingly smaller numbers of muons or kaons.

2. Basic considerations

Double beta decay ($\beta\beta$) is a nuclear transition $(Z, A) \rightarrow (Z + 2, A)$ in which two neutrons bound in a nucleus are simultaneously transformed into two protons plus two electrons (and possibly other light neutral particles). This transition is possible and potentially observable because nuclei with even Z and N are more bound than the odd-odd nuclei with the same $A = N + Z$. Analogous transition of two protons into two neutrons are also, in principle, possible in several nuclei, but phase space considerations give preference to the former.

There are two basic modes of the $\beta\beta$ decay. In the two-neutrino mode ($2\nu\beta\beta$) there are 2 $\bar{\nu}_e$ emitted together with the 2 e^- . It is just an ordinary beta decay of two bound neutrons occurring simultaneously, since the sequential decays are forbidden by the energy conservation law. For this mode, clearly, the lepton number is conserved and this mode of decay is allowed in the standard model of electroweak interactions. It has been repeatedly observed in a number of cases and proceeds with a typical

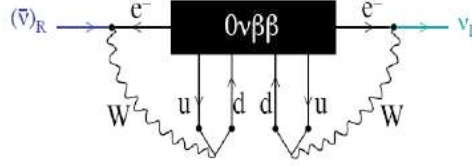


Figure 2: By adding loops involving only standard weak interaction processes the $0\nu\beta\beta$ decay amplitude (the black box) implies the existence of the Majorana neutrino mass.

half-life of $\sim 10^{19-20}$ years. In contrast, in the neutrinoless mode ($0\nu\beta\beta$) only the $2e^-$ are emitted and nothing else. That mode clearly violates the law of lepton number conservation and is forbidden in the standard model. Hence, its observation would be a signal of a "new physics".

One can separate the two modes experimentally by measuring the sum energy of the emitted electrons with a good energy resolution, even if the decay rate for the $0\nu\beta\beta$ mode is much smaller than for the $2\nu\beta\beta$ mode. This is possible since in the $2\nu\beta\beta$ mode the spectrum is continuous, peaked below the $Q/2$ value of the sum kinetic energy, while in the $0\nu\beta\beta$ mode the spectrum is a δ -function at $E_1 + E_2 = Q$, smeared only by the experimental energy resolution (the nuclear recoil is always negligibly small).

The existence of the $0\nu\beta\beta$ decay would mean that on the elementary particle level a six fermion lepton number violating amplitude transforming two d quarks into two u quarks and two electrons is nonvanishing. As was first pointed out by Schechter and Valle³⁾ more than twenty five years ago, this fact alone would guarantee that neutrinos are massive Majorana fermions (see Fig. 2). This qualitative statement (or theorem), however, does not in general allow us to deduce the magnitude of the neutrino mass once the rate of the $0\nu\beta\beta$ decay have been determined. It is important to stress, however, that quite generally an observation of **any** total lepton number violating process, not only of the $0\nu\beta\beta$ decay, would necessarily imply that neutrinos are massive Majorana fermions.

3. Mechanism of the $0\nu\beta\beta$ decay

The rather conservative assumption of how the $0\nu\beta\beta$ decay proceeds is to believe that the only possible way the $0\nu\beta\beta$ decay can occur is through the exchange of a virtual light, but massive, Majorana neutrino (the neutrinos ν_1 , ν_2 and ν_3 of the first

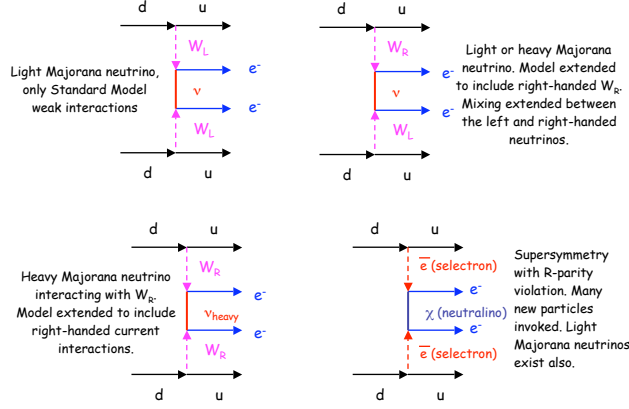


Figure 3: All these symbolic Feynman graphs potentially contribute to the $0\nu\beta\beta$ -decay amplitude

section) between the two nucleons undergoing the transition, and that these neutrinos interact by the standard left-handed weak currents.

If we accept this we can relate the $0\nu\beta\beta$ -decay rate to a quantity containing information about the absolute neutrino mass. With these caveats that relation can be expressed by a well known formula

$$\frac{1}{T_{1/2}^{0\nu}} = G^{0\nu}(Q, Z) |M^{0\nu}|^2 \langle m_{\beta\beta} \rangle^2, \quad (4)$$

where $G^{0\nu}(Q, Z)$ is a phase space factor that depends on the transition Q value and through the Coulomb effect on the emitted electrons on the nuclear charge, and that can be easily and accurately calculated (a complete list of the phase space factors $G^{0\nu}(Q, Z)$ and $G^{2\nu}(Q, Z)$ can be found, e.g. in Ref. ⁴⁾), $M^{0\nu}$ is the nuclear matrix element that can be evaluated in principle, although with a considerable uncertainty and is discussed in detail later, and finally the quantity $\langle m_{\beta\beta} \rangle$ is the effective neutrino Majorana mass, representing the important particle physics ingredient of the process.

In turn, the effective mass $\langle m_{\beta\beta} \rangle$ is related to the mixing angles θ_{ij} (or to the matrix elements $|U_{l,i}|$ of the neutrino mixing matrix) that are determined or constrained by the oscillation experiments, to the absolute neutrino masses m_i of the mass eigenstates ν_i and to the totally unknown additional parameters, as fundamental as the mixing angles θ_{ij} , the so-called Majorana phases $\alpha(i)$,

$$\langle m_{\beta\beta} \rangle = |\sum_i |U_{ei}|^2 e^{i\alpha(i)} m_i|. \quad (5)$$

Here U_{ei} are the matrix elements of the first row of the neutrino mixing matrix.

But that is not the only possible mechanism. LNV interactions involving so far unobserved much heavier (\sim TeV) particles can lead to a comparable $0\nu\beta\beta$ decay rate. Some of the possible mechanisms of the elementary $dd \rightarrow uu + e^-e^-$ transition

(the “black box” in Fig. 2) are indicated in Fig. 3. Only the graph in the upper left corner would lead to the usual relation between the decay rate and neutrino mass, eq. 4. Thus, in the absence of additional information about the mechanism responsible for the $0\nu\beta\beta$ decay one could not unambiguously infer the magnitude of $\langle m_{\beta\beta} \rangle$ from the $0\nu\beta\beta$ -decay rate.

In general $0\nu\beta\beta$ decay can be generated by (i) light massive Majorana neutrino exchange or (ii) heavy particle exchange (see, e.g. Refs.^{5,6}), resulting from LNV dynamics at some scale Λ above the electroweak one. The relative size of heavy (A_H) versus light particle (A_L) exchange contributions to the decay amplitude can be crudely estimated as follows⁷:

$$A_L \sim G_F^2 \frac{\langle m_{\beta\beta} \rangle}{\langle k^2 \rangle}, \quad A_H \sim G_F^2 \frac{M_W^4}{\Lambda^5}, \quad \frac{A_H}{A_L} \sim \frac{M_W^4 \langle k^2 \rangle}{\Lambda^5 \langle m_{\beta\beta} \rangle}, \quad (6)$$

where $\langle m_{\beta\beta} \rangle$ is the effective neutrino Majorana mass, $\langle k^2 \rangle \sim (100 \text{ MeV})^2$ is the typical light neutrino virtuality, and Λ is the heavy scale relevant to the LNV dynamics. Therefore, $A_H/A_L \sim O(1)$ for $\langle m_{\beta\beta} \rangle \sim 0.1 - 0.5 \text{ eV}$ and $\Lambda \sim 1 \text{ TeV}$, and thus the LNV dynamics at the TeV scale leads to similar $0\nu\beta\beta$ -decay rate as the exchange of light Majorana neutrinos with the effective mass $\langle m_{\beta\beta} \rangle \sim 0.1 - 0.5 \text{ eV}$.

Obviously, the lifetime measurement by itself does not provide the means for determining the underlying mechanism. The spin-flip and non-flip exchange can be, in principle, distinguished by the measurement of the single-electron spectra or polarization (see e.g. ⁸). However, in most cases the mechanism of light Majorana neutrino exchange, and of heavy particle exchange, cannot be separated by the observation of the emitted electrons. Thus one must look for other phenomenological consequences of the different mechanisms. Here I discuss the suggestion⁹ that under natural assumptions the presence of low scale LNV interactions, and therefore the absence of proportionality between $\langle m_{\beta\beta} \rangle^2$ and the $0\nu\beta\beta$ -decay rate also affects muon lepton flavor violating (LFV) processes, and in particular enhances the $\mu \rightarrow e$ conversion compared to the $\mu \rightarrow e\gamma$ decay.

The discussion is concerned mainly with the branching ratios $B_{\mu \rightarrow e\gamma} = \Gamma(\mu \rightarrow e\gamma)/\Gamma_\mu^{(0)}$ and $B_{\mu \rightarrow e} = \Gamma_{\text{conv}}/\Gamma_{\text{capt}}$, where $\mu \rightarrow e\gamma$ is normalized to the standard muon decay rate $\Gamma_\mu^{(0)} = (G_F^2 m_\mu^5)/(192\pi^3)$, while $\mu \rightarrow e$ conversion is normalized to the muon nuclear capture rate Γ_{capt} . The main diagnostic tool in our analysis is the ratio

$$\mathcal{R} = B_{\mu \rightarrow e}/B_{\mu \rightarrow e\gamma}, \quad (7)$$

and the relevance of our observation relies on the potential for LFV discovery in the forthcoming experiments MEG ¹⁰ ($\mu \rightarrow e\gamma$) and MECO ¹¹ ($\mu \rightarrow e$ conversion)^a.

^aEven though MECO experiment was recently cancelled, proposals for experiments with similar sensitivity exist elsewhere.

The important quantities are the scales for both LNV and LFV. If they are well above the weak scale, then one would not expect to observe any signal in the forthcoming LFV experiments, nor would the effects of heavy particle exchange enter $0\nu\beta\beta$ at an appreciable level. In this case, the only origin of a signal in $0\nu\beta\beta$ at the level of prospective experimental sensitivity would be the exchange of a light Majorana neutrino, leading to eq.(4), and allowing one to extract $\langle m_{\beta\beta} \rangle$ from the decay rate.

In general, however, the two scales may be distinct, as in SUSY-GUT ¹²⁾ or SUSY see-saw ¹³⁾ models. In these scenarios, both the Majorana neutrino mass as well as LFV effects are generated at the GUT scale. The effects of heavy Majorana neutrino exchange in $0\nu\beta\beta$ are, thus, highly suppressed. In contrast, the effects of GUT-scale LFV are transmitted to the TeV-scale by a soft SUSY-breaking sector without mass suppression via renormalization group running of the high-scale LFV couplings. Consequently, such scenarios could lead to observable effects in the upcoming LFV experiments, but with an $\mathcal{O}(\alpha)$ suppression of the branching ratio $B_{\mu \rightarrow e}$ relative to $B_{\mu \rightarrow e\gamma}$ due to the exchange of a virtual photon in the conversion process rather than the emission of a real one.

The case where the scales of LNV and LFV are both relatively low (\sim TeV) is more subtle and requires more detailed analysis. This is the scenario which might lead to observable signals in LFV searches and at the same time generate ambiguities in interpreting a positive signal in $0\nu\beta\beta$. This is the case where one needs to develop some discriminating criteria.

Based on the analysis in Ref. ⁹⁾, we can formulate the main conclusions regarding the discriminating power of the ratio \mathcal{R} :

1. Observation of both the LFV muon processes $\mu \rightarrow e$ and $\mu \rightarrow e\gamma$ with relative ratio $\mathcal{R} \sim 10^{-2}$ implies, under generic conditions, that $\Gamma_{0\nu\beta\beta} \sim \langle m_{\beta\beta} \rangle^2$. Hence the relation of the $0\nu\beta\beta$ lifetime to the absolute neutrino mass scale is straightforward.
2. On the other hand, observation of LFV muon processes with relative ratio $\mathcal{R} \gg 10^{-2}$ could signal non-trivial LNV dynamics at the TeV scale, whose effect on $0\nu\beta\beta$ has to be analyzed on a case by case basis. Therefore, in this scenario no definite conclusion can be drawn based on LFV rates.
3. Non-observation of LFV in muon processes in forthcoming experiments would imply either that the scale of non-trivial LFV and LNV is above a few TeV, and thus $\Gamma_{0\nu\beta\beta} \sim \langle m_{\beta\beta} \rangle^2$, or that any TeV-scale LNV is approximately flavor diagonal (this is an important caveat).

4. Nuclear matrix elements

Let us assume that the active neutrinos ν_1, ν_2 and ν_3 are indeed massive Majorana fermions. If that is so then the neutrinoless $\beta\beta$ decay will occur and its rate will be

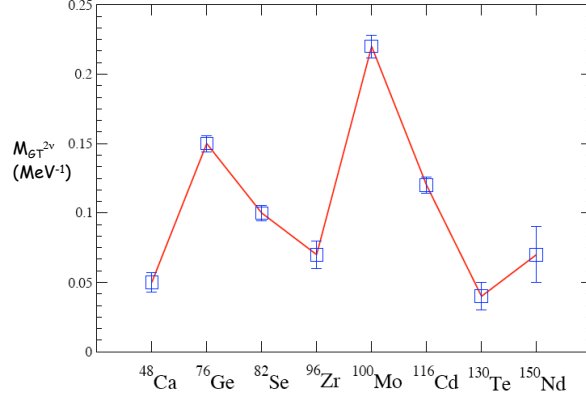


Figure 4: Nuclear matrix elements for the $2\nu\beta\beta$ decay extracted from the measured half-lives.

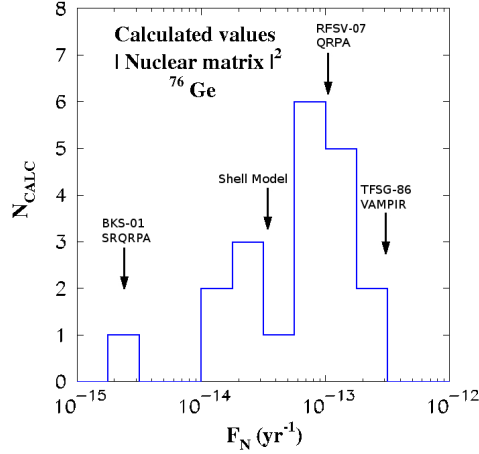
governed by eq.(4). Thus, we need to know the value of the nuclear matrix elements $M^{0\nu}$ in order to plan and interpret the experiments. The nuclear transition involved consists of changing two neutrons, bound in the ground state (always $J^\pi = 0^+$) of the initial even-even nucleus, into two protons bound in the ground state (again always $J^\pi = 0^+$) of the final nucleus. (Here we do not consider the case of the excited final nuclear states.)

In the $2\nu\beta\beta$ decay the two decaying neutrons are uncorrelated. The corresponding momentum transfer from the initial neutron to the final proton is small ($q \sim \text{MeV}$, the momentum of the $e^- + \bar{\nu}_e$, $qR \ll 1$) and thus the long wavelength approximation is valid. Since the isospins of the initial and final nuclei are different, only the Gamow-Teller operator remains and the corresponding nuclear matrix element is

$$M_{GT}^{2\nu} = \sum_m \frac{\langle f || \sigma \tau_+ || m \rangle \langle m || \sigma \tau_+ || i \rangle}{E_m - (M_i + M_f)/2}, \quad (8)$$

where $|m\rangle$ is the set of all $J^\pi = 1^+$ states in the virtual intermediate nucleus. Once the lifetime of the $2\nu\beta\beta$ decay is known, the matrix element $M_{GT}^{2\nu}$ can be easily extracted since $1/T_{1/2} = G^{2\nu}(Q, Z)(M_{GT}^{2\nu})^2$, where the phase space factors $G^{2\nu}(Q, Z)$ can be easily and accurately calculated (see ⁴). In Fig. 4 the $M_{GT}^{2\nu}$ determined in this way are depicted. Note the rapid variation of their value when different nuclei are involved.

For the $0\nu\beta\beta$ decay the situation is quite different. The two initial neutrons, that are transformed into the two final protons, are correlated. The virtual massive Majorana neutrino that connects them has momentum at least of the order of $q > 1/R$ (R is the nuclear radius, as we will show below the actual momentum transfer is even larger) and thus the long wavelength approximation is not valid; all virtual inter-



BKS-01 = A. Bobyk, W. Kaminski, F. Simkovic, PRC**63** (2001) $\Leftarrow 2\nu\beta\beta$ 20 times too slow
 Shell Model = E. Caurier, G. Martinez-Pinedo, F. Nowacki, A. Poves A. Zuker, RMP**77** (2005)
 RFSV-07 = V.R., A. Faessler, F. Simkovic, P. Vogel, NPA**793** (2007) (2003) $\Leftarrow 2\nu\beta\beta$ fitted
 TFSG-86 = T. Tomoda, A. Faessler, K. W. Schmid, F. Grummer, NPA**452** (1986)
 $\Leftarrow 2\nu\beta\beta$ 8 times too fast

Figure 5: Histogram of older published calculated values of $(M^{0\nu})^2$ for ^{76}Ge . The failure of some of the calculations to reproduce the known $2\nu\beta\beta$ -decay lifetime is indicated.

mediate states can in principle contribute. On the other hand, the typical nuclear excitation are less than the energy of the virtual neutrino, hence the “closure approximation” is valid, and we usually do not need to worry about the energies of intermediate states.

In nuclear structure theory one begins with the mean field approximation in which the nucleons are bound in a potential, but independent. That is, however, a poor approximation, and the “residual interaction” need to be taken into account, typically with some truncation. There are two basic and complementary methods of evaluating $M^{0\nu}$, the quasiparticle random phase approximation (QRPA and its various generalizations) and the nuclear shell model (NSM). These two methods differ fundamentally in the way the indicated truncation is implemented. In QRPA one selects a wide interval of single-particle orbits but only a class of configurations (particle-hole and its iterations) of the nucleons are taken into account. On the other hand, in NSM only a relatively narrow interval of single-particle orbits is chosen (one oscillator shell or less) but all (or almost all) configurations of the valence nucleons residing on those orbits are included. In both methods an effective interaction is used, based on the known nucleon-nucleon force, but modified slightly using selected nuclear data as guidance. Since these two methods are so different, it is important to test them against each other.

There are many evaluations of the matrix elements $M^{0\nu}$ in the published literature (for the latest review see ¹⁴). However, the resulting matrix elements often do not

agree with each other and it is difficult, based on the published material, to decide who is right and who is wrong, and what is the theoretical uncertainty in $M^{0\nu}$. That was stressed in the paper by Bahcall *et al.* few years ago¹⁵⁾ where a histogram of 20 calculated values of $(M^{0\nu})^2$ for ^{76}Ge was plotted, with the implication that the width of that histogram is a measure of uncertainty. That is clearly not a valid conclusion as one could see in Fig. 5 where the failure of the outliers to reproduce the known $2\nu\beta\beta$ -decay lifetime is indicated.

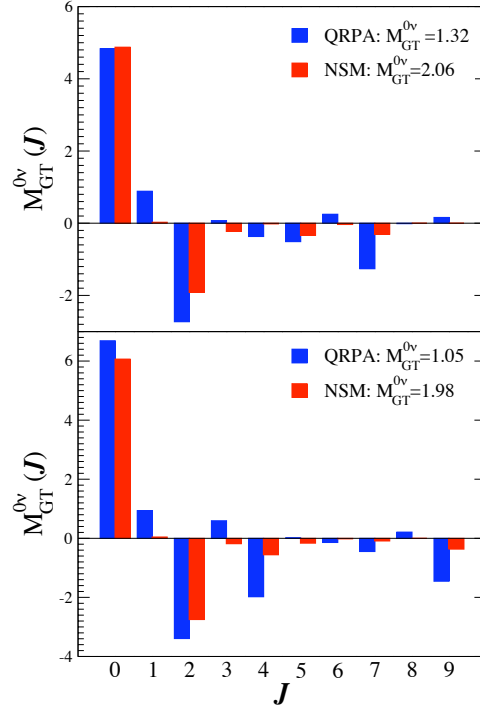


Figure 6: Contributions of different angular momenta \mathcal{J} of the two participating neutrons to the Gamow-Teller part of $M^{0\nu}$ in ^{82}Se (upper panel) and ^{130}Te (lower panel). The results of the nuclear shell model (NSM) (dark histogram) and QRPA treatments (lighter histogram) are compared. Both calculations use the same single-particle spaces: $(f_{5/2}, p_{3/2}, p_{1/2}, g_{9/2})$ for ^{82}Se and $(g_{7/2}, d_{5/2}, d_{3/2}, s_{1/2}, h_{11/2})$ for ^{130}Te . In the QRPA calculation the experimental $2\nu\beta\beta$ -decay rate was used to adjust the strength of the particle-particle interaction.

So why are the calculated values of different authors different? In order to understand the difficulties in evaluating $M^{0\nu}$ we plot in Fig. 6 the contribution of different angular momenta \mathcal{J} of the two transformed neutrons. There are two opposing tendencies in Fig. 6. The large positive contribution (essentially the same in QRPA and NSM) is associated with the so-called pairing interaction of neutrons with neutrons and protons with protons. As the result of that interaction the nuclear ground state is mainly composed of Cooper-like pairs of neutrons and protons coupled to $\mathcal{J} = 0$. The transformation of one neutron Cooper pair into one Cooper proton pair is responsible for the $\mathcal{J} = 0$ piece in Fig. 6.

However, the nuclear hamiltonian contains, in addition, important neutron-proton interaction. That interaction, primarily, causes presence in the nuclear ground state of “broken pairs”, i.e. pairs of neutrons or protons coupled to $\mathcal{J} \neq 0$. Their effect, as seen in Fig. 6, is to reduce drastically the magnitude of $M^{0\nu}$. In treating these terms, the agreement between QRPA and NSM is only semi-quantitative. Since the pieces related to the “pairing” and “broken pairs” contribution are almost of the same magnitude but of opposite signs, an error in one of these two competing tendencies is enhanced in the final $M^{0\nu}$. The competition, illustrated in Fig. 6, is the main reason behind the spread of the calculated $M^{0\nu}$. Many authors use different, and sometimes inconsistent, treatment of the neutron-proton interaction.

In our QRPA (and RQRPA, renormalized QRPA) calculations^{16,17,18)} we adjust the neutron-proton particle-particle interaction, responsible for the “broken pairs” contribution, using the known $2\nu\beta\beta$ -decay lifetimes, respectively the corresponding $M^{2\nu}$ matrix elements. We based this procedure on the fact that the Gamow-Teller strength, the contribution of the 1^+ virtual intermediate states that is fully responsible for the $M^{2\nu}$, is the quantity most sensitive to the corresponding parameter, usually denoted as g_{pp} . The nominal value, corresponding to the G-matrix based on the realistic nucleon-nucleon force, is $g_{pp} = 1$. The renormalized g_{pp} has values between 0.8 and 1.2.

In the NSM there is no analog to adjustment of the g_{pp} parameter. The hamiltonian (primarily its so-called monopole part) is adjusted so that a set of nuclear spectroscopic data is optimally reproduced. That is a complicated and time consuming procedure, but as a result a number of nuclear properties is well described. There is no attempt to reproduce specifically the $2\nu\beta\beta$ -decay lifetime, but the agreement with experiment is, in most cases, acceptable^{19,20,21)}.

To see some additional reasons why different authors obtain in their calculations different nuclear matrix elements we need to analyze the dependence of the $M^{0\nu}$ on the distance r between the pair of initial neutrons (and, naturally, the pair of final protons) that are transformed in the decay process. That analysis reveals, at the same time, the various physics ingredients that must be included in the calculations so that realistic values of the $M^{0\nu}$ can be obtained.

The corresponding $0\nu\beta\beta$ decay operator contains, besides the spin and isospin operators, the “neutrino potentials”, i.e. the Fourier transform of the neutrino propagator. The simplest and most important of these potentials has the form

$$H(r) = \frac{R}{r} \Phi(\omega r) , \quad (9)$$

where R is the nuclear radius introduced here to make the potential, and the resulting $M^{0\nu}$, dimensionless (the $1/R^2$ in the phase space factor compensates for this), r is the distance between the transformed neutrons (or protons) and $\Phi(\omega r)$ is a rather slowly varying function of its argument.

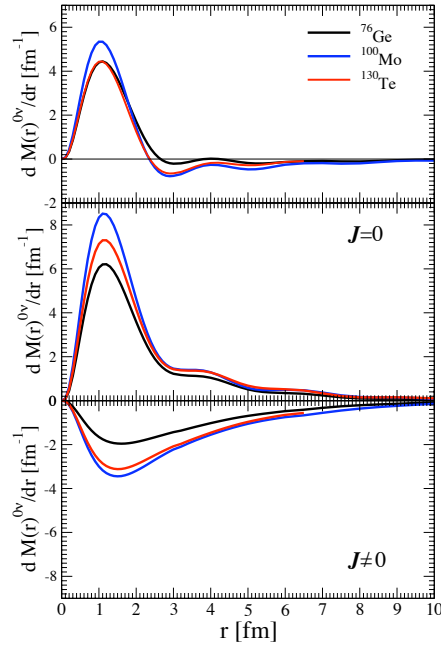


Figure 7: The dependence on r of $M^{0\nu}$ for ^{76}Ge , ^{100}Mo and ^{130}Te . The upper panel shows the full matrix element, and the lower panel shows separately ‘pairing’ ($\mathcal{J} = 0$ of the two participating neutrons) and ‘broken pair’ ($\mathcal{J} \neq 0$) contributions.

From the form of the potential $H(r)$ one would, naively, expect that the characteristic value of r is the typical distance between the nucleons in a nucleus, namely that $\bar{r} \sim R$. However, that is not true as was demonstrated first in Ref. ¹⁸⁾ and illustrated in Fig. 7. One can see there that the competition between the “pairing” and “broken pairs” pieces essentially removes all effects of $r \geq 2 - 3$ fm. Only the relatively short distances contribute significantly. The same result was obtained in the NSM ²¹⁾. (We have also shown in ¹⁸⁾ that an analogous result is obtained in an exactly solvable, semirealistic model. There we also showed that this behaviour is restricted to an interval of the parameter g_{pp} that contains the realistic value near unity.)

Once the r dependence displayed in Fig. 7 is accepted, several new physics effects come to mind. One of them is the short-range nucleon-nucleon repulsion known from scattering experiments. Two nucleon strongly repel each other at distances $r \leq 0.5 - 1.0$ fm, i.e. the distances very relevant to evaluation of the $M^{0\nu}$. The nuclear wave functions used in QRPA and NSM, products of the mean field single-nucleon wave function, do not take into account the influence of this repulsion that is irrelevant in most standard nuclear structure theory applications. The standard way to include the effect is to modify the radial dependence of the $0\nu\beta\beta$ operator so that the effect of short distances (small values of r) is reduced.

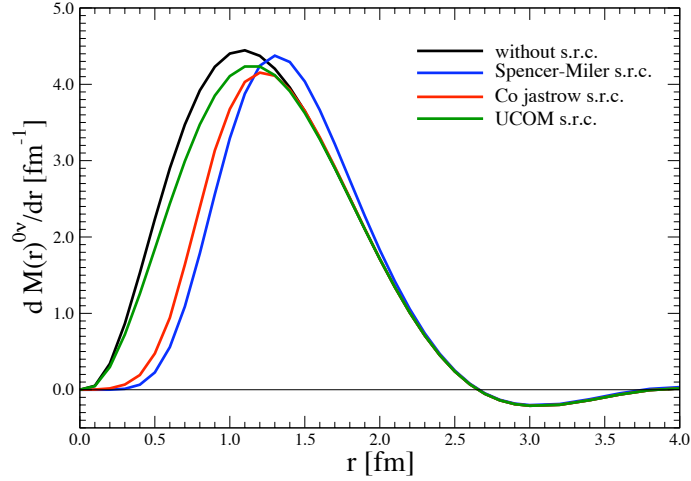


Figure 8: The r dependence of $M^{0\nu}$ in ^{76}Ge . The four curves show the effects of different treatments of short-range correlations. The resulting $M^{0\nu}$ values are 5.32 when the effect is ignored, 5.01 when the UCOM transformation is applied and 4.14 when the treatment based on the Fermi hypernetted chain and 3.98 when the phenomenological Jastrow function is used. (See the text for details.)

Traditionally, this was done by multiplying the neutrino potential by the square of a Jastrow-like function first derived in ²²⁾ and in a more modern form in ²³⁾. That phenomenological procedure reduces the magnitude of $M^{0\nu}$ by 20-25% as illustrated in Fig. 8. Recently, another procedure, based on the Unitary Correlation Operator Method (UCOM) has been proposed ²⁴⁾. That procedure, still applied not fully consistently, reduces the $M^{0\nu}$ much less, only by about 5% ²⁵⁾. It is prudent to include these two possibilities as extremes and the corresponding range as systematic error. Once a consistent procedure is developed, consisting of deriving an effective $0\nu\beta\beta$ decay operator that includes (probably perturbatively) the effect of the high momentum (or short range) that component of the systematic error could be substantially reduced.

Another effect that needs to be taken into account is the nucleon finite size. That is included, usually, by introducing the dipole form of the nucleon form factor

$$f_{V,A} = \left(\frac{1}{1 + q^2/M_{V,A}^2} \right)^2, \quad (10)$$

where the cut-off parameters $M_{V,A}$ have values (deduced in the reactions of free neutrinos with free or quasifree nucleons) ~ 1 GeV. This corresponds to the nucleon size of $\sim 0.5 - 1.0$ fm. Note that in our case we are dealing with neutrinos far off mass shell, and bound nucleons, hence it is not obvious that the above form factors are applicable. It turns out, however, that once the short range correlations are properly included (by either of the procedures discussed above) the $M^{0\nu}$ becomes essentially independent of the adopted values when $M_{V,A} \geq 1$ GeV. However, in the past various authors neglected the effect of short range correlations, and in that case

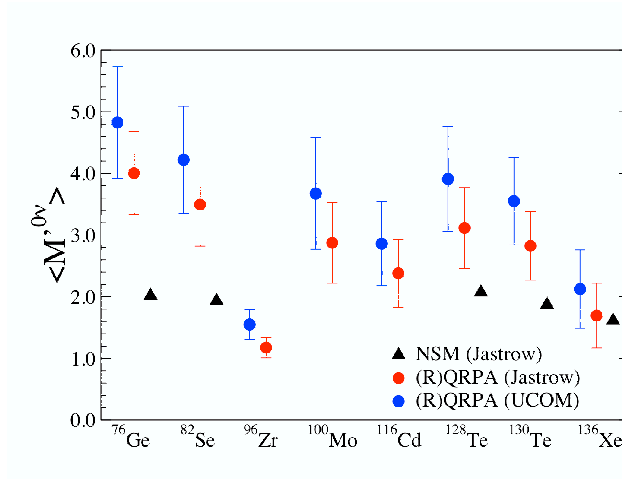


Figure 9: The full ranges of $M'^{0\nu}$ with the two alternative treatments of the short range correlations. For comparison the results of a recent Large Scale Shell Model evaluation of $M'^{0\nu}$ that used the Jastrow-type treatment of short range correlations are also shown (triangles).

a proper inclusion of nucleon form factor (or their neglect) again causes variations in the calculated $M^{0\nu}$ values.

Yet another correction that various authors neglected must be included in a correct treatment. Since $r \leq 2\text{-}3$ fm is the relevant distances, the corresponding momentum transfer $1/r$ is of the order of ~ 200 MeV, much larger than in the ordinary β decay. Hence the induced nucleon currents, in particular the pseudoscalar (since the neutrinos are far off mass shell) give noticeable contributions^{18,26)}.

We have, therefore, identified the various physics effects that ought to be included in a realistic evaluation of $M^{0\nu}$ values. The spread of the calculated values, noted by Bahcall *et al.*¹⁵⁾ can be often attributed to the fact that various authors either neglect some of them, or include them inconsistently.

5. Discussion and conclusion

Even though we were able to explain, or eliminate, a substantial part of the spread of the calculated values of the nuclear matrix elements, sizeable systematic uncertainty remains. That uncertainty, within QRPA and RQRPA, as discussed in Refs.^{16,17)}, is primarily related to the difference between these two procedures, to the size of the single-particle space included, whether the so-called quenching of the axial current coupling constant g_A is included or not, and to the systematic error in the treatment of short range correlations¹⁸⁾. In Fig. 9 the full ranges of the resulting matrix elements $M^{0\nu}$ is indicated. The indicated error bars are highly correlated; e.g., if true values are near the lower end in one nucleus, they would be near the lower ends in all indicated nuclei.

The figure also shows the most recent NSM results ²⁰⁾. Those results, obtained with Jastrow type short range correlation corrections, are noticeably lower than the QRPA values. That difference is particularly acute in the lighter nuclei ⁷⁶Ge and ⁸²Se. While the QRPA and NSM agree on many aspects of the problem, in particular on the role of the competition between “pairing” and “broken pairs” contributions and on the r dependence of the matrix elements, the disagreement in the actual values remains to be explained.

When one compares the 2ν and 0ν matrix elements (Figs. 4 and 9) the feature to notice is the fast variation in $M^{2\nu}$ when going from one nucleus to another while $M^{0\nu}$ change only rather smoothly, in both QRPA and NSM. This is presumably related to the high momentum transfer (or short range) involved in $0\nu\beta\beta$. That property of the $M^{0\nu}$ matrix elements makes the comparison of results obtained in different nuclei easier and more reliable.

While substantial progress has been achieved, we are still somewhat far from being able to evaluate the $0\nu\beta\beta$ nuclear matrix elements confidently and accurately. Perhaps at the next Venice meeting we will be able to report that we reached that goal.

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